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Axial Vibrations of a Whirling Bar

26 FEBRUARY 1962

Prepared by P. G. BHUTA

Prepared for DEPUTY COMMANDER AEROSPACE SYSTEMS
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Inglewood, California

LABORATORIES DIVISION • AEROSPACE CORPORATION
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ABSTRACT

In this paper axial vibrations of whirling bars are studied using undeformed coordinates. It is found that, in general, the effect of whirling is to lower the natural frequencies of the bars. For the static case an interesting result, which has not been previously reported, is obtained as a special case of the dynamic problem. When the angular velocity of the bar approaches certain critical values, "static resonances" occur and the axial displacements everywhere in the bar tend to become unboundedly large. These resonances are explained physically. The same problem is also worked out using final or deformed coordinates and it is shown that the static resonances cannot come out of such an analysis. Moreover, the rotation does not have any effect on the natural frequencies when deformed coordinates are used. The results of the analysis using undeformed coordinates appear to be more compatible with the physics of the problem.

I. INTRODUCTION

The study of whirling bars is of interest because they find application as machine elements such as turbines, propellers and helicopter blades. More recently, they have been employed as antenna elements of artificial satellites used for communications purposes. In this paper axial vibrations of whirling bars are studied using undeformed coordinates. It is found that, in general, the effect of whirling is to lower the natural frequencies of the bars. The advantage of using undeformed or initial coordinates is that they yield, for the static case, an interesting result as a special case of the dynamic problem. This result for the static case has not been previously reported. When the angular velocity of the bar, ω, approaches certain critical values, ω_i, static resonances occur and the axial displacements everywhere in the bar tend to become unboundedly large. These resonances are explained physically. The same problem is also resolved using final or deformed coordinates. Such a resonance can never result from an analysis using deformed coordinates since, for the axial displacement, u, to get arbitrarily large, the deformed coordinate must become arbitrarily large. Thus, the only u that can become arbitrarily large is the one for which the deformed coordinate becomes large, and the resonant character of the solution is lost. As the angular velocity of the bar approaches zero, the frequency equations reduce to those of bars that vibrate axially but are otherwise at rest. A comparison of the static solutions obtained from the use of deformed and undeformed coordinates is made. Moreover, the use of undeformed coordinates does not even show the effect of rotation on the natural frequencies.

II. DIFFERENTIAL EQUATIONS OF MOTION

Let (X, Y) be a reference system fixed in space and (x, y) a coordinate system attached to the undeformed whirling bar with its origin coincident

with that of the (X, Y) system. The (x, y) coordinate system will then have an angular velocity ω (for a fixed x) which will be called the angular velocity of the bar. If the displacement vector is defined as the difference between the radius vector to the particle at time, t, denoted by $\vec{R}$, and the radius vector to the particle at some initial time, t_o, denoted by $\vec{R}_o$, then the displacement vector, $\vec{V}$, is given by $\vec{R}-\vec{R}_o$ and the acceleration vector by $\vec{a} = \frac{\partial^2 \vec{V}}{\partial t^2}$ ([1], p. 72). Now $\vec{V}$ as it stands could be quite large, and convective terms must be used in differentiating it with respect to time, since small deformation theory cannot be applied ([2], p. 85). This would result in a nonlinear theory, which is inconvenient. To eliminate the use of convective terms, it is assumed that the particle does not deform greatly from the undeformed rotating position (the x, y system rotates with the angular velocity ω). Define a displacement $\vec{v}$ such that $\vec{V} = \vec{r} + \vec{v}$, where $\vec{r}$ is the position vector to the rotating point located at x. The acceleration will then be given by $\vec{a} = \frac{\partial^2 (\vec{r}+\vec{v})}{\partial t^2}$. To carry out this process, set in a unit vector $\vec{e}_x$ in the x direction and $\vec{e}_y$ in the y direction. Then the angular velocity vector $\vec{\omega}$ is given by

$$\vec{\omega} = \omega \vec{e}_y \quad , \tag{1}$$

and

$$\vec{r} = (x + u)\, \vec{e}_x \quad , \tag{2}$$

where u is the magnitude of the component of $\vec{v}$ in the x direction.

The other components of $\vec{v}$ in the direction of the tangent in the plane of rotation, and in a direction transverse to the plane of rotation, are ignored since only the axial vibrations of whirling bars are considered in this

paper. One of this paper's objects is to bring out the advantage of using undeformed or initial coordinates in this problem. Carrying out the differentiation indicated earlier, the equation of motion, employing the usual assumptions for the one-dimensional elastic wave propagation in bars, is

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} - \rho\omega^2(x+u) \quad , \tag{3}$$

where σ is the normal stress in the x direction and ρ is the density of the material. To obtain equilibrium equations, deformed coordinates should be used. However, in the limit of small strain theory (which assumption is made here), the equilibrium equations and the stresses become identical ([1], Chap. 5) for both systems of coordinates. The stress-strain relation is

$$\sigma = E \frac{\partial u}{\partial x} \quad . \tag{4}$$

Introducing (4) into (3) yields

$$\frac{\partial^2 u}{\partial x^2} - \alpha^2 \frac{\partial^2 u}{\partial t^2} + \alpha^2\omega^2 u = - \alpha^2\omega^2 x \quad , \tag{5}$$

where

$$\alpha^2 = \frac{\rho}{E} \quad , \tag{6}$$

and E is the modulus of elasticity.

III. BOUNDARY CONDITIONS

Two sets of boundary conditions will be considered. The first set may pertain to the vibrations of a helicopter blade or a communications satellite antenna, whereas the second set pertains to the web of a flywheel with a rigid flange on the outside.

(a) The boundary conditions for the first case are:

$$u(0,t) = 0 \quad , \tag{7a}$$

$$\frac{\partial u(\ell,t)}{\partial x} = 0 \quad , \tag{7b}$$

which correspond to the case of a fixed-free bar.

(b) The boundary conditions for the second case are:

$$u(0,t) = 0 \quad , \tag{8a}$$

$$u(\ell,t) = 0 \quad , \tag{8b}$$

which correspond to the case of a fixed-fixed bar.

IV. SOLUTIONS OF BOUNDARY VALUE PROBLEMS

For the solution of the boundary value problem, split u such that

$$u(x,t) = u_1(x) + u_2(x,t) \quad , \tag{9}$$

where u_1 and u_2 satisfy the following

$$\frac{d^2u_1}{dx^2} + \alpha^2\omega^2 u_1 = -\alpha^2\omega^2 x \quad , \tag{10}$$

and

$$\frac{\partial^2 u_2}{\partial x^2} + \alpha^2\omega^2 u_2 = \alpha^2 \frac{\partial^2 u_2}{\partial t^2} \quad . \tag{11}$$

The general solution of (10) gives the solution for the static problem and may at once be written as

$$u_1 = a_1 \sin \alpha\omega x + a_2 \cos \alpha\omega x - x \quad . \tag{12}$$

A solution of (11) may be taken as

$$u_2(x,t) = U(x)\, e^{jp_n t} \quad , \tag{13}$$

where p_n is the frequency to be determined. Substitution of (13) into (11) results in

$$\frac{d^2U}{dx^2} + \alpha^2(\omega^2 + p_n^2)\, U = 0 \quad . \tag{14}$$

The general solution of (14) is

$$U = a_3 \sin \alpha\sqrt{\omega^2+p_n^2}\, x + a_4 \cos \alpha\sqrt{\omega^2+p_n^2}\, x \quad . \tag{15}$$

Case (a): fixed-free bar

Since the boundary conditions (7) have to be satisfied for all values of time, it follows that u_1 and u_2 each have to satisfy (7). It may be

remarked that the static solution for u_1, as given by (12), differs from the one given by Biezeno and Grammel ([3], p. 124) in that final coordinates are used in [3], whereas the use of initial coordinates in the present paper has resulted in a different differential equation. Application of (7a) and (7b) results in

$$a_2 = 0 \quad , \tag{16}$$

and

$$a_1 = \frac{1}{\alpha\omega \cos \alpha\omega\ell} \quad , \tag{17}$$

provided that ω does not correspond to a root of

$$\cos \alpha\omega\ell = 0 \quad . \tag{18}$$

For the present it will be assumed that ω does not correspond to a root of (18). The case when ω corresponds to a root of (18) will be considered separately. Hence the static solution is

$$u_1 = \frac{\sin \alpha\omega x}{\alpha\omega \cos \alpha\omega\ell} - x \quad . \tag{19}$$

For the case when ω corresponds to one of the roots of (18), a solution to (10), satisfying the boundary conditions (7a) and (7b), does not exist. When one encounters such nonexistence of a solution to a physical problem it must have significance. The physical problem must have some sort of a solution even though it only tends toward infinity as ω approaches ω_i, one of the critical values [roots of (18)] . As ω approaches ω_i, the constant a_1

given by (17) becomes very large. Such values of ω_i are given by

$$\omega_i = \left(\frac{2i+1}{2}\right)\frac{\pi}{\ell}\sqrt{\frac{E}{\rho}} \quad , \quad i=0,1,2\ldots \qquad . \tag{20}$$

Thus, the lowest angular velocity at which the static resonance occurs in a fixed-free bar is the velocity at which the peripheral speed of the bar is equal to $\frac{\pi}{2}$ times the speed of sound in the material of the bar. At such speeds the solution for u_1 becomes large for all values of x between zero and ℓ. Hence, one can say that, for any value of x between zero and ℓ, u_1 becomes unboundedly large as ω approaches ω_i. This is evidently some sort of a resonance phenomenon and is directly a result of the use of undeformed coordinates in the present analysis. At first sight it would appear that u_1 cannot become arbitrarily large, since there is no steady energy input to make it do so. However, this reasoning is fallacious. Consider the bar as being undeformed but having an angular velocity ω at some instant of time t_o. Suppose that at this time the bar is allowed to deform. The axial displacement for the steady state or static case, u_1, will immediately begin to increase, hopefully to attain some steady value. As u_1 increases, the moment of inertia will increase, since the elemental mass will now be located at a distance $x+u_1$ instead of at x. The angular momentum of the bar will then increase, since ω is assumed to be constant, and energy must be added to the system. It is not unreasonable to suppose that there will be some value of ω such that this process does not lead to a steady bounded value of u_1, and the roots of (18) are just such values of ω. Actually, only the lowest value of ω_i is of consequence since it is unlikely that equilibrium physically can be attained for ω greater than the lowest value of ω_i.

For u_2, the application of boundary conditions (7a) and (7b) to (15) results in

$$a_4 = 0 \tag{21}$$

and

$$a_3 \alpha \sqrt{\omega^2+p_n^2} \cos \alpha \sqrt{\omega^2+p_n^2}\, \ell = 0 \tag{22}$$

For a nontrivial solution it is required that

$$\alpha \sqrt{\omega^2+p_n^2}\, \ell = (2n+1)\frac{\pi}{2}, \quad n=0,1,2,\ldots \tag{23}$$

From (23) the frequency equation can be rewritten as

$$p_n^2 = \left(\frac{2n+1}{2}\right)^2 \frac{\pi^2 E}{\ell^2 \rho} - \omega^2, \quad n=0,1,2,\ldots \tag{24}$$

If ω is allowed to approach zero, (24) reduces to the frequency equation of a fixed-free bar that is vibrating axially, but is otherwise at rest. Equation (24) shows that, in general, the effect of whirling is to lower the natural frequencies. It is of interest to consider two special cases arising from (24) as ω increases. First, it is possible for ω to be large enough so that the right member of (24) vanishes for a certain value of n. It can easily be shown by referring to (15) that this corresponds to a static instability in the particular n^{th} mode. It should be remarked that this static instability is the

same as the one which has already been mentioned for u_1. Second, consider the case when ω is allowed to be large enough so that the right member of (24) becomes negative for values of $n < n_c$ and is positive for $n > n_c$. Then for $n > n_c$, vibrations exist whereas, for $n < n_c$, p_n becomes imaginary and these modes could get damped out or grow asymptotically with time depending on the sign. However, it is not meaningful to consider the vibrations of the bar in this domain, since it is unlikely that static equilibrium can be attained for such values of ω_i.

Case (b): fixed-fixed bar

Application of boundary conditions (8) to (12) gives

$$a_2 = 0 \quad , \tag{25}$$

and

$$a_1 = \frac{\ell}{\sin \alpha\omega\ell} \quad , \tag{26}$$

provided that ω does not correspond to a root of

$$\sin \alpha\omega\ell = 0 \quad . \tag{27}$$

Hence,

$$u_1 = \frac{\ell}{\sin \alpha\omega\ell} \sin\alpha\omega x - x \quad . \tag{28}$$

When ω corresponds to a root of (27); i. e.,

$$\omega_i = \frac{i\pi}{\ell}\sqrt{\frac{E}{\rho}} \quad , \quad i=1, 2, 3, \ldots \quad . \tag{29}$$

static resonances occur, as has already been explained in the case of a fixed-free bar. The lowest angular velocity at which the static resonance occurs in a fixed-fixed bar is the velocity at which the peripheral speed of the bar is equal to π times the speed of sound in the material of the bar. Only the lowest value of ω_i is of interest as has been remarked in the case of a fixed-free bar.

Similarly, when applied to u_2, the boundary conditions (8a) and (8b) yield

$$a_4 = 0 \quad , \tag{30}$$

and

$$a_3 \sin \alpha \sqrt{\omega^2 + p_n^2}\, \ell = 0 \quad . \tag{31}$$

For a nontrivial solution it is required that

$$\alpha \sqrt{\omega^2 + p_n^2}\, \ell = n\pi \ , \quad n=1,2,3,\ldots \quad , \tag{32}$$

from which the frequencies for the fixed-fixed bar are

$$p_n^2 = \frac{n^2\pi^2}{\ell^2} \frac{E}{\rho} - \omega^2 \ , \quad n=1,2,3,\ldots \quad . \tag{33}$$

If ω approaches zero in (33), the resulting equation for p_n is the same as the one for a fixed-fixed bar that is vibrating axially but is otherwise at rest. Remarks pertinent to the ones for the fixed-free bar apply also here, when ω becomes successively large.

V. SOLUTION USING DEFORMED COORDINATES

If now deformed or final coordinates are used in obtaining the inertial forces, the equation of motion (5) becomes

$$\frac{\partial^2 u}{\partial x^2} - \alpha^2 \frac{\partial^2 u}{\partial t^2} = - \alpha^2 \omega^2 x \quad . \tag{34}$$

Split u such that

$$u(x,t) = u_1(x) + u_2(x,t) \quad , \tag{35}$$

where u_1 and u_2 satisfy the following

$$\frac{d^2 u_1}{dx^2} + \alpha^2 \omega^2 x = 0 \quad , \tag{36}$$

and

$$\frac{\partial^2 u_2}{\partial x^2} - \alpha^2 \frac{\partial^2 u_2}{\partial t^2} = 0 \quad . \tag{37}$$

The general solution of (36) is the solution for the static problem and may be written as

$$u_1 = a_1 x + a_2 - \frac{\alpha^2 \omega^2 x^3}{6} \quad . \tag{38}$$

Equation (38) is identical to the one that can be deduced from [3]. For the fixed-free bar the static solution, after evaluating the constants of integration using (7a) and (7b), is

$$u_1 = \frac{\alpha^2 \omega^2 x}{2} \left(\ell^2 - \frac{x^2}{3} \right) . \tag{39}$$

Equation (39) shows that the resonant character of the static solution is indeed lost when deformed coordinates are used. Furthermore, it follows from (37) that the frequency equation for a whirling bar that is vibrating axially would be independent of the angular velocity, ω , with which the bar is whirled. A steady input of energy can be made into the system free of any dissipation without an instability ever occurring, a result which does not appear to be realizable.

VI. NUMERICAL EXAMPLE

To illustrate the results of the analysis, numerical examples were worked out for a steel bar 100 inches in length. The static displacements for a fixed-free bar, as given by deformed and undeformed coordinates, are given in Fig. 1. The ratio of the displacement at the tip of the fixed-free bar obtained from the use of undeformed and deformed coordinates is plotted as a function of ω in Fig. 2. The example shows that for small values of ω the two results do not differ appreciably, but the results of the analysis using undeformed coordinates indicate a new limit to the speed at which the bars can be whirled even with an exotic material having a very high yield point. Figures 3 and 4 [obtained from (24) and (33)] show the effect of whirling on the natural frequencies of fixed-free and fixed-fixed bars, respectively. The intersection of the curve and the abscissa indicates the occurrence of instability.

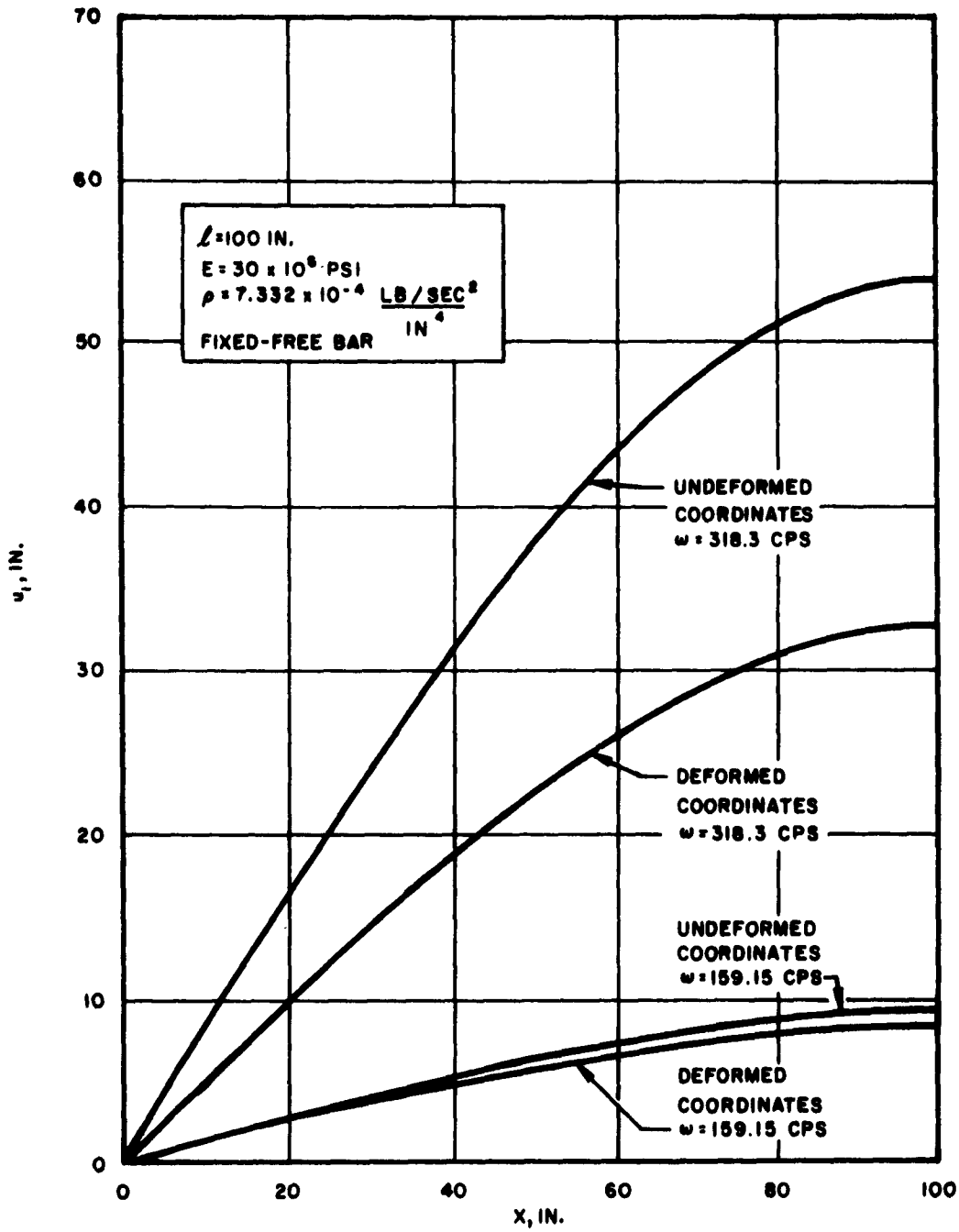

Fig. 1 - Comparison of Static Solutions

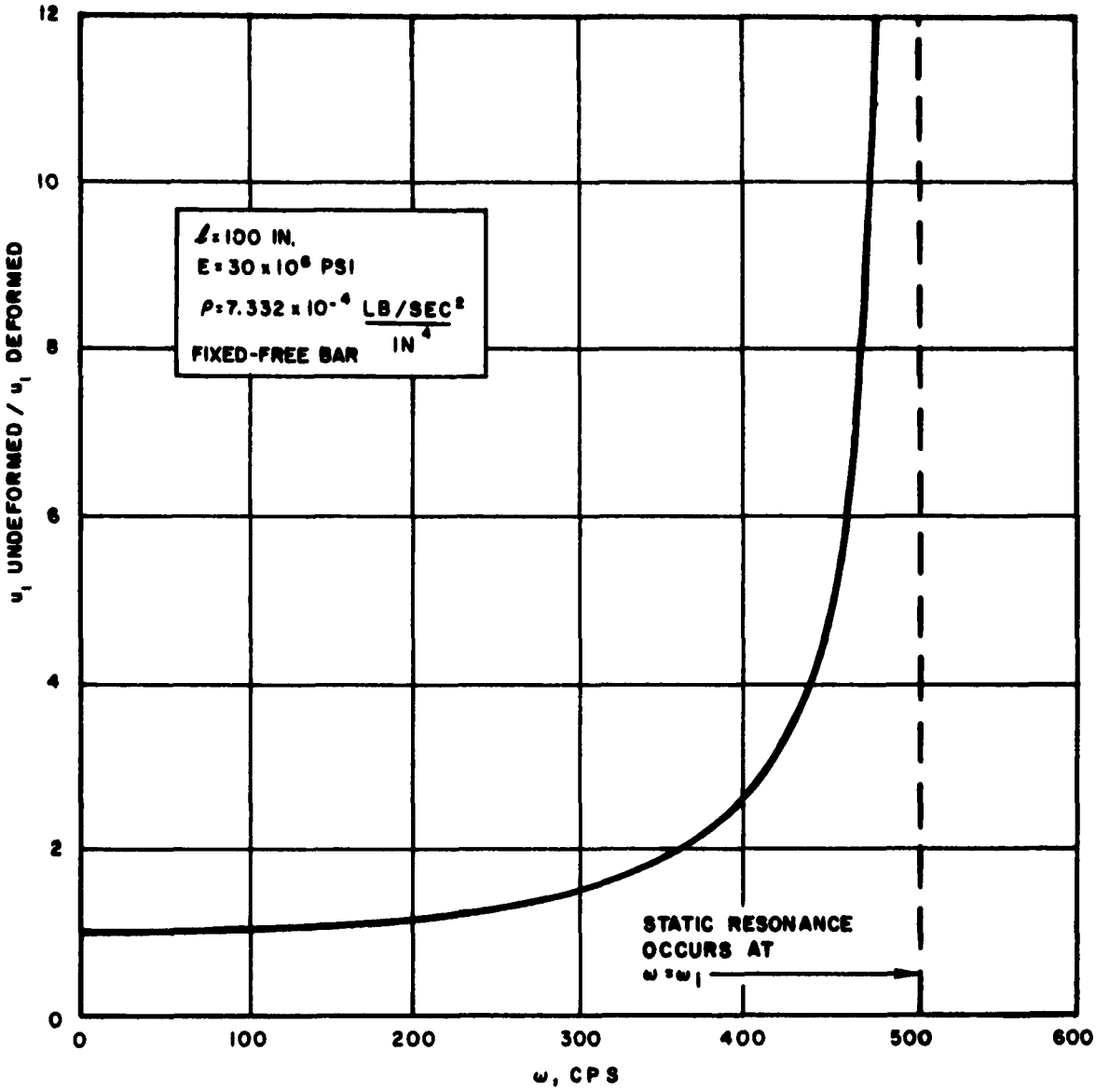

Fig. 2 - Ratio of the Displacements at the Tip of the Bar vs ω

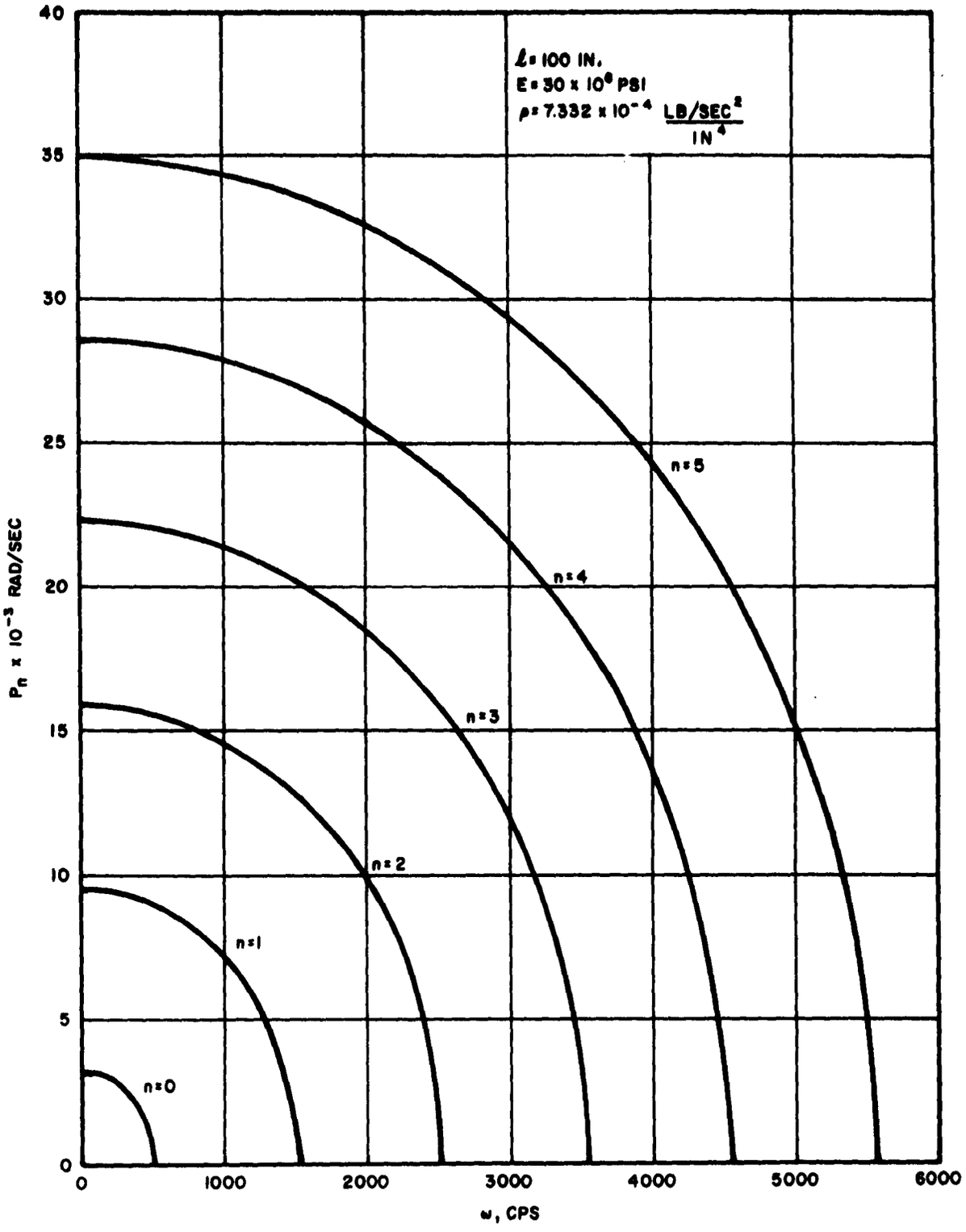

Fig. 3 - Plots of p_n vs ω for a Fixed-Free Bar

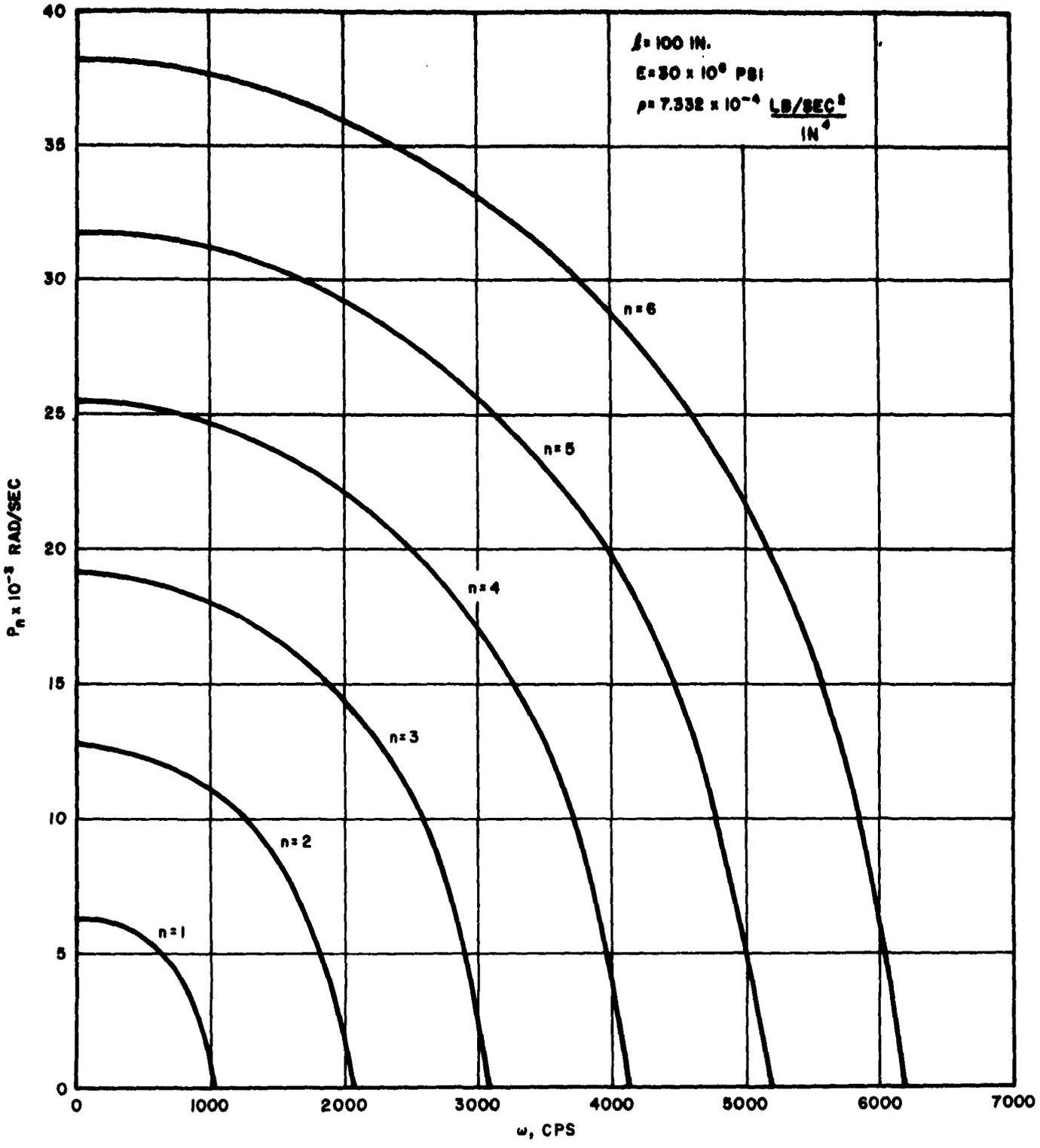

Fig. 4 - Plots of p_n vs ω for a Fixed-Fixed Bar

VII. CONCLUSION

The effect of whirling on the axial vibrations of a bar has been studied in this paper. Certain advantages of the use of undeformed coordinates in this problem are indicated and the results of such an analysis appear to be more compatible with the physics of the problem. When the angular velocity of the bar reaches certain critical values, static resonances occur. These resonances point to a new limit to which the bars can be whirled even with the use of an exotic material having a very high yield point.

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